

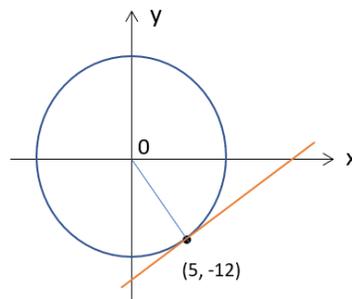
Examination Papers of the 2022 Entrance Exam, UPTP

- *Write your answers in the answer sheet.*
 - *Some formulae are provided in the last page. (Page7)*
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1. A jeweler has five rings, each ring weighing 36 g, made of an alloy of 10% silver and 90% gold. She decides to melt down the rings and add enough silver to reduce the gold content to 75%. How much silver should she add?
(A) 32 g (B) 36 g (C) 40 g (D) 44 g (E) 48 g.

2. Find the center and radius R of an equation of the circle $x^2 + y^2 - 2x - 6y - 15 = 0$.
(A) $(-2, -6)$, $R = 5$ (B) $(-1, -3)$, $R = 5$ (C) $(1, -3)$, $R = 5$ (D) $(-1, 3)$, $R = 5$ (E) $(1, 3)$, $R = 5$.

3. Find an equation for the line tangent to the circle $x^2 + y^2 = 169$ at the point $(5, -12)$.



(A) $5x - 12y = 169$ (B) $5x - 12y = 144$ (C) $12x - 5y = 169$ (D) $12x - 5y = 144$ (E) $5x + 12y = 169$.

4. Boyle's Law states that when a sample of a gas is compressed at a constant temperature, the pressure of the gas is inversely proportional to the volume of the gas. Suppose the pressure of a sample of air that occupies 0.20 m^3 at 25°C is 60 k Pa. If the sample expands to a volume of 0.5 m^3 , find the new pressure.

(A) 20 (B) 24 (C) 28 (D) 32 (E) 36 k Pa.

5. A function is given,

$$f(x) = 1 + 2x^2; x = a, x = a + h.$$

What is the average rate of change between the given values of the variable?

(A) $2a + 2h$ (B) $2a + 4h$ (C) $4a + 2h$ (D) $4a + 4h$ (E) none of the above.

6. A function is given, $y = 1 - 3(x - 3)^2$,

What are the indicated transformations of $f(x) = x^2$ are applied to its graph in the given order?

- (A) shifted 3 units to the left, reflected in the x-axis and stretched by a factor of 3, and shifted upward 1 unit.
 (B) shifted 3 units to the right, reflected in the y-axis and stretched by a factor of 3, and shifted upward 1 unit.
 (C) shifted 3 units to the right, reflected in the x-axis and stretched by a factor of 3, and shifted downward 1 unit.
 (D) shifted 3 units to the right, reflected in the x-axis and stretched by a factor of 3, and shifted upward 1 unit.
 (E) none of the above.

7. A function is given,

$$f(x) = \frac{3x + 2}{x - 2}.$$

What is the inverse function of $f(x)$?

(A) $f^{-1}(x) = \frac{3x+3}{x+2}$ (B) $f^{-1}(x) = \frac{3x-3}{-x+2}$ (C) $f^{-1}(x) = \frac{2x-2}{x-2}$ (D) $f^{-1}(x) = \frac{2x-2}{x+3}$ (E) $f^{-1}(x) = \frac{2x+2}{x-3}$.

8. The number of apples produced by each tree in an apple orchard depends on how densely the trees are planted. If n trees are planted on an acre of land, then each tree produce $600 - 6n$ apples. The number of apples produced per acre is

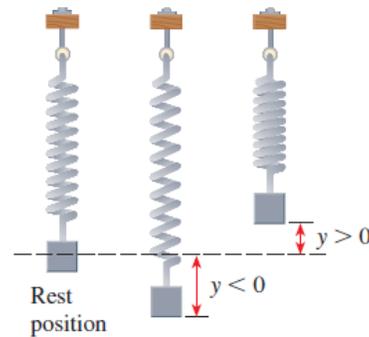
$$A(n) = n(600 - 6n).$$

How many trees should be planted per acre to obtain maximum yield of apples?

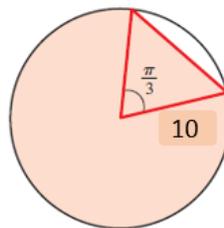


- (A) 40 (B) 45 (C) 50 (D) 55 (E) 60.
9. A polynomial is given, $p(x) = x^3 - 3x^2 - 4x$.
What are the zeros of the polynomial?
(A) 0, -1, 4 (B) 0, 1, 4 (C) 0, -1, -4 (D) 0, 1, -4 (E) none of above.
10. Let $P(x) = 2x^3 - 7x^2 + 8$. Find the quotient $Q(x)$ and remainder $R(x)$ when $P(x)$ is divided by $x - 3$.
(A) $Q(x) = 2x^2 - x + 3, R(x) = 1$ (B) $Q(x) = 2x^2 + x - 3, R(x) = -1$ (C) $Q(x) = 2x^2 - x + 3, R(x) = -1$
(D) $Q(x) = 2x^2 - x - 3, R(x) = 1$ (E) $Q(x) = 2x^2 - x - 3, R(x) = -1$.
11. Find a polynomial of degree 3 and zeros 2 and $1 + i$.
(A) $P(x) = x^3 - 4x^2 + 6x - 4$ (B) $P(x) = x^3 - 4x^2 - 6x + 4$ (C) $P(x) = x^3 - 2x^2 + 4x - 2$
(D) $P(x) = x^3 - 2x^2 - 4x + 2$ (E) $P(x) = x^3 - 2x^2 + 4x - 1$.
12. After simplification, the expression $3 \ln x + \frac{1}{2} \ln y - 3 \ln(y^2 + 1)$ can be reduced to
(A) $\ln\left(\frac{3x\sqrt{y}}{(y^2+1)^3}\right)$ (B) $\log\left(\frac{3x\sqrt{y}}{(y^2+1)^3}\right)$ (C) $\ln(x^3\sqrt{y} - 3(y^2 + 1))$ (D) $\ln\left(\frac{x^3\sqrt{y}}{(y^2+1)^{1/3}}\right)$ (E) $\ln\left(\frac{x^3\sqrt{y}}{(y^2+1)^3}\right)$.
13. Solve the logarithmic equation $4 + 2 \log(2x) = 10$. $x = ?$
(A) 50 (B) 100 (C) 500 (D) 1000 (E) 5000.
14. A car engine runs at a temperature of 92°C , when the engine is turned off, it cools according to Newton's Law of Cooling ($T = T_s + D_0 e^{-kt}$) with constant $k = 0.04$, where the time is measured in minutes. How much time does it take for the engine to cool to 38°C if the surrounding temperature is 20°C .
(A) 35min (B) 40min (C) 45min (D) 50min (E) 55min.
15. The decibel level B , measured in decibels (dB), is defined as $B = 10 \log(I/I_0)$, where I_0 is a reference sound intensity $I_0 = 10^{-12} \text{ W/m}^2$. Find the intensity level of a jet engine at takeoff if the decibel level was measured at 130 dB.
(A) $I = 10^{-2} \text{ W/m}^2$ (B) $I = 10^{-1} \text{ W/m}^2$ (C) $I = 10^0 \text{ W/m}^2$ (D) $I = 10^1 \text{ W/m}^2$ (E) $I = 10^2 \text{ W/m}^2$.
16. Regarding the signs of the trigonometric functions, which one of the following descriptions is 'NOT' correct?
(A) sin function is positive in quadrant II (B) sec function is negative in quadrant II (C) tan function is positive in quadrant III (D) tan function is negative in quadrant IV (E) none of the above.

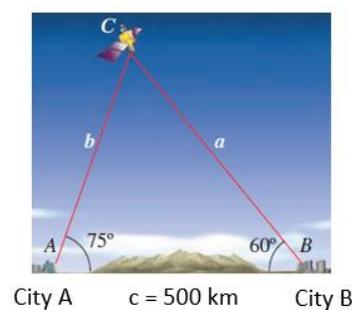
17. The displacement of a mass suspended by a spring is modeled by the function $y = 10 \sin 10\pi t$, where y is measured in centimeters (cm) and t in seconds (sec). Which one of the following descriptions is 'NOT' correct?



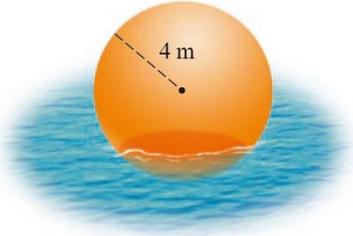
- (A) amplitude is 10 cm (B) period is 0.5 sec (C) frequency is 5 cycles per second
 (D) the object is in simple harmonic motion (E) none of the above.
18. Suppose the radius of the circle is 10, and the central angle is $60^\circ(\pi/3)$, then the area of the shaded region is



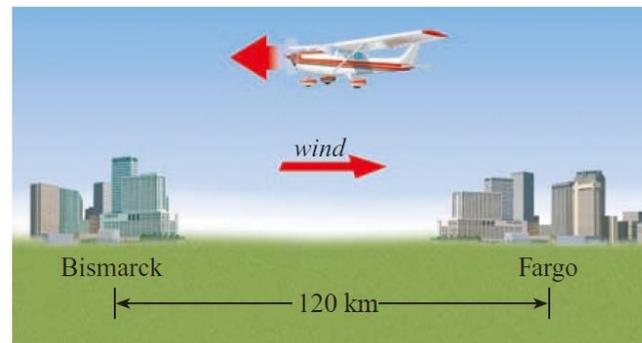
- (A) $\frac{200\pi}{3} + 25\sqrt{3}$ (B) $\frac{250\pi}{3} + 20\sqrt{3}$ (C) $\frac{250\pi}{3} + 25\sqrt{3}$ (D) $\frac{250\pi}{3} + 30\sqrt{3}$ (E) none of the above.
19. Find $\cos\left(\sin^{-1}\frac{4}{5}\right)$.
- (A) $-\frac{3}{4}$ (B) $-\frac{3}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$ (E) none of the above.
20. A satellite orbiting the earth passes directly overhead at observation stations in City A and City B, 500 km apart. At an instant when the satellite is between these two stations, its angle of elevation is simultaneously observed to be 75° in City A and 60° in City B, as shown in the figure. How far is the satellite from City A?



- (A) $\frac{250\sqrt{2}}{\sqrt{3}}$ (B) $\frac{500\sqrt{2}}{\sqrt{3}}$ (C) $\frac{250\sqrt{3}}{\sqrt{2}}$ (D) $\frac{500\sqrt{3}}{\sqrt{2}}$ (E) none of the above.
21. Please find the simple expression of $\frac{\sec x - \cos x}{\sec x}$.
- (A) $\sin^2 x$ (B) $\cos^2 x$ (C) $\sec^2 x$ (D) $\csc^2 x$ (E) none of the above.

22. What is the exact value of the expression $\sin\left(\cos^{-1}\frac{\sqrt{3}}{2} - \sec^{-1}\frac{2\sqrt{3}}{3}\right)$?
- (A) $\frac{\sqrt{3}}{2}$ (B) $\sqrt{3}$ (C) $\frac{1}{2}$ (D) 0 (E) none of the above.
23. Please find the identity of $\frac{\sin 5x - \sin x}{\cos 5x + \cos x}$.
- (A) $\tan x$ (B) $\cot x$ (C) $\tan 3x$ (D) $\cot 3x$ (E) none of the above.
24. Which one is 'NOT' the solution of $2 \sin^2 \theta - \cos \theta = 1$?
- (A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) π (D) $\frac{5\pi}{3}$ (E) 3π .
25. Please express the $x^2 = 8y$ in the polar coordinates.
- (A) $r = 8 \cos \theta \sin \theta$ (B) $r = 8 \sec \theta \csc \theta$ (C) $r = 8 \csc \theta \tan \theta$ (D) $r = 8 \sec \theta \tan \theta$ (E) none of the above.
26. Regarding the symmetric characteristics of the polar graph of the function $r=3\sec\theta$, which of the followings is valid about the graph?
- (A) symmetric about the pole (B) symmetric about the polar axis
(C) symmetric about the $\theta = \pi/2$ axis (D) not symmetric about the polar axis
(E) does not have any symmetric feature.
27. Please find the power of complex number $(-\sqrt{2} - \sqrt{2}i)^5$. (note: Using De Moivre's Theorem)
- (A) $2\sqrt{2} + 2\sqrt{2}i$ (B) $4\sqrt{2} + 4\sqrt{2}i$ (C) $16\sqrt{2} + 16\sqrt{2}i$ (D) $32\sqrt{2} + 32\sqrt{2}i$ (E) $64\sqrt{2} + 64\sqrt{2}i$.
28. A description of a line is given. The line crosses the z-axis where $z = 5$ and crosses the xy-plane where $x = 2, y = 3$. Please find the parametric equations for the line.
- (A) $x = -2t, y = -3t, z = -5t$ (B) $x = -2t, y = -3t, z = 5 - 5t$
(C) $x = 2t, y = 3t, z = 5 - 5t$ (D) $x = 2t, y = 3t, z = 5 + 5t$ (E) $x = 2t, y = 3t, z = -5t$.
29. Please find the projection of \mathbf{u} onto \mathbf{v} if $\mathbf{u}=\langle 1, 2 \rangle$ and $\mathbf{v}=\langle 1, -3 \rangle$.
- (A) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (B) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (C) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (D) $\left(-\frac{3}{2}, \frac{1}{2}\right)$ (E) $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.
30. A spherical buoy of radius 4 meters floats in a calm lake. 80 cm of the buoy are submerged. Place a coordinate system with the origin at the center of the sphere. Please find an equation of the circle formed at the waterline of the buoy.
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- (A) $x^2 + y^2 = 1.2^2$ (B) $x^2 + y^2 = 2.4^2$ (C) $x^2 + y^2 = 3.6^2$ (D) $x^2 + y^2 = 4.8^2$ (E) $x^2 + y^2 = 5.6^2$.
31. Please find the area of the triangle ΔPQR with vertices $P(1, 0, 2)$, $Q(0, 2, 4)$ and $R(1, 3, 5)$.
- (A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{3\sqrt{2}}{2}$ (D) $\frac{3\sqrt{3}}{2}$ (E) $\frac{5\sqrt{2}}{2}$.

32. A man flies a small airplane from Fargo to Bismarck, North Dakota - a distance of 120 km. Because he is flying into a headwind, the trip takes him 2h. On the way back, the wind is still blowing at the same speed, so the return trip takes only 1h and 12 min. Which of the following answer is correct?



- (A) Wind Speed = 20 km/h (B) Wind Speed = 23.5 km/h (C) Wind Speed = 30 km/h
 (D) Airplane Speed = 40 km/h (E) Airplane Speed = 73.5 km/h.

33. Please find inverse matrix of $\begin{bmatrix} 2 & 4 & 1 \\ -1 & 1 & -1 \\ 1 & 4 & 0 \end{bmatrix}$.

- (A) $\begin{bmatrix} 4 & -4 & -5 \\ 1 & 1 & -1 \\ -5 & 4 & 6 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 4 & 5 \\ -1 & 1 & -1 \\ 5 & -4 & -6 \end{bmatrix}$ (C) $\begin{bmatrix} -4 & -4 & 5 \\ 1 & 1 & -1 \\ 5 & 4 & -6 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & -4 & 5 \\ 1 & -1 & -1 \\ -5 & 4 & -6 \end{bmatrix}$ (E) No inverse matrix.

34. Use Cramer's Rule to solve the system of equations. Which answer is 'NOT' correct?

$$\begin{cases} x - y + 2z = 0 \\ 3x + y + z = 11 \\ -x + 2y - z = 0 \end{cases}$$

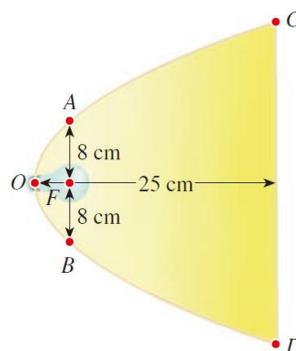
- (A) $|D| = 9$ (B) $|D_x| = 33$ (C) $|D_y| = -11$ (D) $|D_z| = -11$ (E) none of the above.

35. A rational function can be expressed by partial fraction as shown below, which of the following answer is correct?

$$r(x) = \frac{4x^2 + 8}{x^3 - 2x^2 - 8x} = \frac{A}{x} + \frac{B}{x + 2} + \frac{C}{x - 4}$$

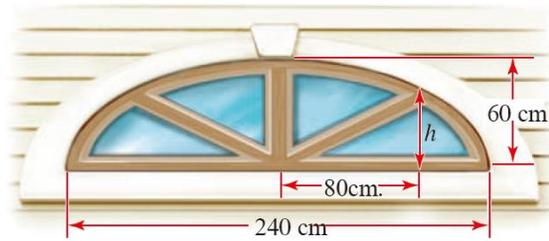
- (A) (A, B, C) = (1, 2, 3) (B) (A, B, C) = (-1, -2, -3) (C) (A, B, C) = (-1, -2, 3) (D) (A, B, C) = (1, -2, -3)
 (E) (A, B, C) = (-1, 2, 3).

36. A lamp with a parabolic reflector is shown in the figure below. The bulb is placed at the focus, and the focal diameter is 16 cm. Please find the diameter $d(C, D)$ of the opening, 25 cm from the vertex in parabolic reflector.



- (A) 10 (B) 20 (C) 40 (D) 60 (E) 80.

37. A “sunburst” window above a doorway is constructed in the shape of the top half of an ellipse, as shown in the figure below. The window is 60 cm tall at its highest point and 240 cm wide at the bottom. Find the height (h) of the window 80 cm from the center of the base.



- (A) $5\sqrt{5}$ (B) $10\sqrt{5}$ (C) $15\sqrt{5}$ (D) $20\sqrt{5}$ (E) none of the above.
38. Which of the following equation is a hyperbolic equation?
 (A) $3x^2 + 4y^2 - 6x - 24y + 39 = 0$ (B) $9x^2 - y^2 + 18x + 6y = 0$
 (C) $4x^2 - y^2 - 8x + 2y + 3 = 0$ (D) $16x^2 - 9y^2 - 96x + 288 = 0$ (E) none of the above.
39. An architect designs a theater with 12 seats in the first row, 18 in the second, 24 in the third, and so on. If the theater is to have a seating capacity of 1380, how many rows must the architect use in his design?
 (A) 20 (B) 21 (C) 22 (D) 23 (E) none of the above.
40. Please find the sum $\frac{1}{3} + \frac{2}{3^2} + \frac{2^2}{3^3} + \frac{2^3}{3^4} + \dots + \frac{2^9}{3^{10}}$
 (A) $\frac{18659}{19683}$ (B) $\frac{19171}{19683}$ (C) $\frac{57001}{59049}$ (D) $\frac{58025}{59049}$ (E) $\frac{175099}{177147}$.